LINEAR PROGRAMMING

CASE STUDIES

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Case study 1

Portfolio selection

- Select a portfolio package from a set of alternative investments
- Maximization of the expected return or minimization of the risk
- Available capital
- Company's policy
- Duration of investments' economic life, potential growth rate, danger, liquidity

Return data

Expected annual return of investments

Investment	Expected annual return rate (%)
Share A – manufacturing sector	15.4
Share B - manufacturing sector	19.2
Share C - food and beverage sector	18.7
Share D – food and beverage sector	13.5
Mutual fund E	17.8
Mutual fund Z	16.3

Requirements

- Total amount = € 90000
- Amount in shares of a sector no larger than 50% of total available
- Amount in shares with the larger return of a sector less or equal to 80% of sector's total amount
- Amount in manufacturing company B less or equal to 10% of the whole share amount
- Amount in mutual funds less or equal to 25% of the amount in manufacturing shares

Solution

Decision variables

x_1 = invested amount in share A of the manufacturing sector

x_2 = invested amount in share B of the manufacturing sector

x_3 = invested amount in share C of the food and beverage sector

x_4 = invested amount in share D of the food and beverage sector

x_5 = invested amount in mutual fund E

x_6 = invested amount in mutual fund Z

<u>**Case Study 2**</u>

Financial programming problem

- Initial amount: € 80000
- Timeframe of investments' decisions: 4 months
- Two-month government bonds: return 3%
- Three-month government bills: return 6.5%
- Bank deposits: interest 1%
- At the beginning of the 5^{th} month at least € 40000 are needed
- Maximum amount in two-month or three-month bonds: € 32000

Decision variables

B_j = amount to be invested in government bonds at the beginning of the month j

C_j = amount to be invested in government bills at the beginning of the month j

D_j= amount to be invested in bank deposits at the beginning of the month j

Case Study 3

Investment choice problem in a limited capitals status

- There are five independent investments
- Maximization of the total present net value
- Satisfaction of budget limitations
- Cash inflows of the investments
- Cash outflows of the investments
- Each investment is divisible (investment rate)

Inflow data

Cash inflows

Year	Investment				
	1	2	3	4	5
1	38	11	17	8	25
2	41	16	24	11	28
3	54	15	29	13	35
4	-	20	-	19	46

Outflow data

Cash outflows

Year	Investment					Amount Available
	1	2	3	4	5	
0	34	10	16	9	31	55
1	13	5	8	4	10	28
2	14	6	10	6	13	30
3	17	6	11	7	12	37
4	-	8	-	5	16	30

Case study 4

Advertising media selection

Problem data

Advertising media	Cost of one view (in €)	Units of expected audience rate of one view
1. Friday – day	400	5000
2. Saturday – day	450	5500
3. Sunday – day	450	5700
4. Friday – night	500	7500
5. Saturday – night	550	8200
6. Sunday - night	550	8400

Other relative elements

- Goal: Determination of views / records in order to maximize the total audience rate
- Total available amount: € 45000
- Maximum amount for Friday: € 11000
- Maximum amount for Saturday: € 14400
- Total daily view number: at least 20
- Total nightly view number: at least 50% of the total
- Maximum view number: each day 12, each night 18

Decision variables

X_1 = number of views on Friday (day)

X_2 = number of views on Saturday (day)

X_3 = number of views on Sunday (day)

X_4 = number of views on Friday (night)

X_5 = number of views on Saturday (night)

X_6 = number of views on Sunday (night)

Case study 5

Marketing research

- Personal interviews, Daily (D) and Nightly (N)
- Households: with children, without children, of one person
- Sample = 800 households
- At least: 400 with children, 200 without children, 100 of one person
- Respondents Night $\geq$ Respondents Day
- To be done during: At least 50% of interviews to households with children
  At least 60% of interviews to households without children
  At least 70% of interviews to households of one person

Cost elements

Interview cost (in monetary units)

Household category	Day	Night
With children	1500	1800
Without children	1300	1600
One person	1000	1200

Decision variables

x_{11} = Number of interviews to households with children carried out during the day

x_{12} = Number of interviews to households with children carried out during the night

x_{21}= Number of interviews to households without children carried out during the day

x_{22} = Number of interviews to households without children carried out during the night

x_{31} = Number of interviews to households of one person carried out during the day

x_{32} = Number of interviews to households of one person carried out during the night

Case study 6

Human resources management

- Allocation of available human resources to different departments, work centers, shifts etc.

- Recruitment of seasonal staff

- Allocation of staff to shifts

- Minimizing the number of employees who should work in various time periods during the day

Problem's data

Time period	Shift					Minimum number of required employees
	1	2	3	4	5	
07 a.m. – 09 a.m.	+					35
09 a.m. – 11 a.m.	+	+				68
11 a.m. – 13 p.m.	+	+				60
13 p.m. – 15 p.m.	+	+	+			57
15 p.m. – 17 p.m.		+	+	+		65
17 p.m. – 19 p.m.			+	+		63
19 p.m. – 21 p.m.			+	+		72
21 p.m. – 23 p.m.				+		33
23 p.m. – 07 a.m.					+	12
Gross employee cost per day (in €)	230	220	225	240	260	

Case study 7

Production planning

- Planning horizon: A number of time periods
- Problem: Having a forecast for each period's demand, determine the products' quantities that can be produced with feasible methods in order to satisfy the total demand with the minimum cost.
- For two products A and B, there are demand forecasts for January, February and March.
- Initial stock: 100 units of product A and 120 units of product B.
- Minimum total required stock: 130 units of product A and 110 of product B
- Unit production cost: A = € 20 and B = € 25
- Maintenance cost per period and per unit: 2% on the unit production cost

Demand

Bicycle demand

Month	Bicycle	
	A	B
January	700	800
February	900	600
March	1000	900
Total	2600	2.300

Capacity

System's capacity

Month	Machine capacity (machine hours)	Available work (man-hours)
January	3000	2500
February	2800	2300
March	3600	2400

Use of resources

Required resources per product unit

Bicycle	Machine hours	Man-hours
A	1.5	1.1
B	1.6	1.2

Decision variables

x_{IJ} = number of units of product I produced during month J,
where I = A, B and J = 1, 2, 3

I_{IJ} = number of units of product I maintained in stock at the end of the month J,
where I = A, B and J = 1, 2, 3
J = 1 – January
J = 2 – February
J = 3 – March

Case study 8

Diet problem

- Identification of a diet or of a prescription meeting specific dietary requirements
- Criterion: minimum cost

X_j = the amount of ingredient j for the production of one unit of animal feed

Problem's data

Required nutritional ingredient	Number of nutritional ingredients per ingredient unit				Nutritional requirement per animal feed unit
	Ingredient				
	1	2	3	4	
Vitamin A	80	115	100	90	>= 80
Vitamin C	110	90	85	100	>= 100
Vitamin E	50	70	105	80	>= 60
Proteins	250	300	210	240	>= 260
Calories	480	510	470	530	<= 2300
Unit cost	180	160	145	200	

Case study 9

Mix problem

- Determination of the best mix program of raw material for the production of final products
- Three main ingredients A, B and C
- Three products: super fuel, unleaded, super unleaded
- Minimum required octane number
- Maximization of the total daily profit
- Available quantities of main ingredients
- Minimum required product quantities

Problem's data

Main ingredient	Octane number	Cost per ton (€)	Maximum daily available quantity (tones)
A	120	38	1000
B	90	42	1200
C	130	105	700

Demand data

Fuel	Octane number	Cost per ton (€)	Daily demand (tones)
A	94	85	800
B	92	80	1100
C	96	88	500

Decision variables

X_{ij} = quantity of ingredient i mixed for the production of one tone of product j

for i = A, B, C and j = 1, 2, 3
j = 1 – fuel super
j = 2 – unleaded fuel
j = 3 – super unleaded fuel