Delay Distribution Analysis of the RTS/CTS mechanism of IEEE 802.11

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Abstract

A simple and accurate packet delay distribution analysis is presented for the RTS/CTS access scheme of the IEEE 802.11 Distributed Coordination Function (DCF). The packet delay distribution is effectively studied by developing an analytical model that calculates the probability that a packet is successfully transmitted after experiencing a delay time equal to a given value. The model uses simple mathematical relations and provides accurate delay distribution curves. The accuracy of the analytical model is verified by simulations.

1. Introduction

IEEE 802.11 is the standard protocol used in Wireless Local Area Networks (WLANs). IEEE 802.11 specifies Medium Access Control (MAC) and Physical (PHY) layers for WLANs [1]. IEEE 802.11 MAC specifies a primary mechanism called Distributed Coordination Function (DCF) to access the medium. DCF makes use of the Carrier Sense Multiple Access with Collision Avoidance (CSMA/CA) scheme and describes two techniques to transmit data packets; a two-way handshaking (DATA - ACK) called basic access and an optional four-way handshaking (RTS - CTS - DATA - ACK) called Request-To-Send/Clear-To-Send (RTS/CTS) access method. Our model is based on the latter, which has been proved to provide better performance for data [2] and voice traffic [10].

A lot of research has been conducted in analyzing different performance metrics of DCF. Bianchi [2] proposed a 2-dimensional Markov chain model to calculate the throughput performance of the IEEE 802.11 DCF considering infinite number of packet retransmissions. Wu [3] modified the model of [2] taking into account the packet's retransmission limit specified in the standard [1]. Chatzimisios [6] developed a model that calculates the average packet delay. In [7] we suggested a model that calculates the packet delay for a specific a number of retransmissions i.e. packet delay per stage. Although [7] gives a more inside view of packet delays, it also works with average values as [6]. Utilizing *Z*-transform of the packet delay, [4],[5] and [8] calculated the probability generating function (PGF) which in turn is used to compute the probability distribution function (PDF) of the packet delay. The method employed by [4], [5] and [8] is computationally very expensive. The author in [9] proposed an effective model that computes the probability of a packet to be successful transmitted with delay time lower than a given value. Although less complex than [4], [5], and [8] the method of [9] still involves a number of relatively expensive operations.

In this paper we extend [3] and present a simple and very accurate mathematical model to calculate the probability that a packet is successfully transmitted with a delay time equal to a given value. By using our model, delay distribution values can easily be obtained. The main strengths of the model proposed here are simplicity (the model is based on simple mathematical relations), reduced computational complexity (the model is based on simple mathematical operations resulting in short computational times) and accuracy (analytical results almost perfectly match the simulation results). The model covers only the RTS/CTS mode but it could be easily adapted to the basic access mode.

The rest of the paper is organized as follows. Section 2 briefly describes DCF. Section 3 describes the Markov chain used by our model. Section 4 develops the proposed delay distribution model.

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Section 5 validates the model by comparing analytical results against simulations. Finally, Section 6 presents the conclusions.

2. IEEE 802.11 DCF MAC

According to the standard [1], a station transmits its packet if it senses the channel idle for a period of Distributed Inter Frame Space (DIFS). If the channel is sensed busy, the station defers transmission until an idle DIFS is detected and then generates a random backoff interval before transmitting. The backoff time counter is decreased by one at each time slot as long as the channel is sensed idle, it is stopped when the channel is busy and it is resumed when the channel is sensed idle again for more than DIFS. With the RTS/CTS mechanism, a station transmits a short RTS packet first instead of the data packet when its backoff timer reaches zero. The receiving station responds with a CTS packet after a SIFS time interval. The sender is allowed to transmit the data packet only if it receives a valid CTS. Upon the successful reception of the data packet the receiver transmits an ACK frame. If the transmitting station does not receive a CTS frame from the receiver it schedules a retransmission.

The backoff time counter is chosen uniformly in the range $[0, W_j\text{-}1]$, where W_j is the current contention window and j is the backoff stage i.e. the number of packet's unsuccessful transmissions. The contention window at the first transmission of a packet is set equal to $W_{min}=W=W_o$. After an unsuccessful packet transmission the contention window is doubled up to a maximum value $W_{max}=2^m W$ (where m is the number of contention window sizes). Once the contention window reaches W_{max} it remains at this value until it is reset. The contention window is reset to W_{min} after a successful packet transmission or if the packet's retransmission limit R is reached.

3. DCF Mathematical Model

In this study we assume that the channel conditions are ideal (no transmission errors, no hidden stations and no capture effect), a fixed number n of contending stations and that each station has always a packet available for transmission (i.e., the station works in saturated conditions) of the same fixed size l.

Let $b(t)$ and $s(t)$ be the stochastic processes representing the backoff time counter and the backoff stage $(0,\ldots,R)$ respectively for a given station at time t.

The two-dimensional process $\{s(t), b(t)\}$ is a discrete-time Markov chain. In this study we utilize the Markov chain model of [3]. The key approximation in that model is that each packet transmission collides with constant and independent probability p regardless of the backoff stage. The probability τ that a station transmits a packet in a randomly chosen slot time can be expressed as [3]:

$$\tau=\begin{cases} \dfrac{2\cdot(1-2p)\cdot(1-p^{R+1})}{W\cdot(1-(2p)^{R+1})\cdot(1-p)+(1-2p)\cdot(1-p^{R+1})}, & R\leq m \\[2ex] \dfrac{2\cdot(1-2p)\cdot(1-p^{R+1})}{W\cdot(1-(2p)^{m+1})\cdot(1-p)+(1-2p)\cdot[(1-p^{R+1})+W\cdot 2^m\cdot p^{m+1}\cdot(1-p^{R-m})]}, & R>m \end{cases} \qquad (1)$$

The probability p that a transmitted packet encounters a collision is given by:

$$p = 1-(1-\tau)^{n-1} \qquad (2)$$

Equations (1) and (2) represent a non-linear system with two unknown values, τ and p, which can be solved using numerical methods and has a unique solution.

Let q_{tr} be the probability with that at least one station (out of n) transmits in a considered slot time:

$$q_{tr} = 1-(1-\tau)^n \qquad (3)$$

Let q_s be the probability that a transmission occurring on the channel is successful. This can be computed as the probability that only one station transmits and the n-1 remaining stations do not, given the condition that a transmission occurs on the channel. Probability q_s is given by:

$$q_s = \frac{n\cdot\tau\cdot(1-\tau)^{n-1}}{q_{tr}} = \frac{n\cdot\tau\cdot(1-\tau)^{n-1}}{1-(1-\tau)^n} \qquad (4)$$

Let $E[slot]$ be the average length of a slot time. $E[slot]$ is given by:

$$E[slot] = (1-q_{tr})\sigma + q_{tr}q_sT_s + q_{tr}(1-q_s)T_c \qquad (5)$$

where σ is the duration of an empty slot, and T_s and T_c are the time durations the channel is sensed busy during a successful transmission and a collision, respectively.

The time duration of T_s and T_c depends on the channel access method employed. For the RTS/CTS access method (which is the focus of this paper), we have:

$$T_s = DIFS+RTS+SIFS+\delta+CTS+SIFS+\delta+\left(\frac{H+l}{C}\right)+SIFS+\delta+\left(\frac{ACK}{C}\right)+\delta$$

$$T_c = DIFS+RTS+SIFS+CTS$$

where H is the packet header (equal to the sum of MAC and physical headers), l is the packet length, δ is the propagation delay and C is the channel bit rate.

The delay distribution analysis provided in the next section is based on the probabilities τ and p computed above.

4. Delay Distribution

The packet delay is defined as the time interval elapsed between the moment the packet is at the head of the MAC queue and the time when an ACK for this packet is received. The packet delay distribution is a discrete distribution whose the smallest time unit of the backoff counter is one time slot [5].

The first step of our mathematical model is to group packet time delays according to the number of collisions suffered by a packet. Note that this number corresponds to the stage at which the successful transmission occurs [7].

The probability P that a packet is successfully transmitted after suffering a given delay time D is the sum of the probabilities for delay D at all stages [7]:

$$P(D) = \sum_{j=0}^{R} P_j(D) \quad \text{for } 0 \le D \le \infty \tag{6}$$

where R is the retry limit, j is the stage number and P_j is the probability that a packet that is successfully transmitted at stage j suffers a delay equal to D.

The delay D of packet successfully transmitted at backoff stage j takes a value of the form:

$$D = T_s N_s + T_c N_c + N_e \sigma + jT_c + T_s \quad \text{for } 0 \le j \le R \tag{7}$$

where N_e is the number of empty slots that a packet encounters before its successful transmission, σ the duration of an empty slot, N_s (N_c) is the number of successful (collided) transmissions from the rest of the stations that a packet encounters before its successful transmission, jT_c is the time that the packet occupies the channel with collisions until it reaches the j stage and T_s is the time to transmit successfully from the jth stage.

The key assumption upon which we base our analysis is the following one: we assume this the average relationship between N_s, N_c and N_e is a fixed one equal to the average, i.e.

$$N_e = \frac{P_e}{P_s} N_s = \frac{P_e}{P_c} N_c \tag{8}$$

Note that, in average, for every P_{tr} slot times with a transmission (successful P_s or unsuccessful P_c), we have P_e empty slot times, where P_{tr} is the probability with that at least one station out of n-1 transmits in a considered slot time, and P_e is the probability that the channel is idle and are given by:

$$P_{tr} = 1-(1-\tau)^{n-1} \text{ , and } P_e = 1 - P_{tr}$$

$$P_s = (n-1)\cdot\tau\cdot(1-\tau)^{n-2} / P_{tr} \text{ and } P_c = P_{tr} - P_s \tag{9}$$

From the above, we can express D as a function of two variables: N_s and N_c.

$$D = N_s T_s + N_c T_c + (P_e / P_s) N_s \sigma + jT_c + T_s \tag{10}$$

If we can compute the probability that a packet successfully transmitted at backoff stage j encounters N_s successes and N_c collisions, with the above formula we can obtain the probability that this packets suffers a delay D, and therefore we can compute the packets delay distributions. We denote this probability by $P_j(N_s,N_j)$.

Let us start by computing $P_j(N_s,N_0)$ for backoff stage $j = 0$, i.e. $P_0(N_s,N_0)$. Let i be the value of the backoff counter at stage 0 in a single station. The value i is decremented by one after a transmission slot time (T_s or T_c) or after an idle slot time (σ). The probability Γ_i that a packet with backoff value i is successfully transmitted from stage 0 after experiencing N_s successful and N_0 unsuccessful transmissions is computed as:

$$\Gamma_i = \frac{1-p}{1-p^{R+1}} \cdot \frac{1}{W_0} \cdot P_s^{N_s} P_c^{N_0} P_e^{i-N} \cdot C \quad \text{for } 0 \le N \le i \tag{11}$$

where N=N_s+N_0, $1/W_o$ is the probability that the backoff counter gets the value of i, the $(1-p)/(1-p^{R+1})$ is the probability that the packet is successfully transmitted and not dropped, and $P_s^{N_s} P_c^{N_0} P_e^{i-N}$ is the probability that the packet experiences N_s successful and N_0 unsuccessful transmissions in i slots, and C are all the combinations of N_s and N_0 transmissions in i backoff slots:

$$C = \frac{i!}{N_s! N_0! (i-N)!} \quad \text{for } 0 \le N \le i \tag{12}$$

As the backoff counter can get any value from 0 to W-1 in stage 0, the probability of a successfully transmitted packet from stage 0 that experiences Ns and N_0 transmissions is

$$P_0(N_s, N_0) = \sum_{i=N}^{W_0-1} \Gamma_i \quad \text{for } 0 \le N \le W_0 - 1 \tag{13}$$

From (11), (12) and (13) we get:

$$P_0(N_s, N_0) = A_0 \cdot H_0 \cdot \sum_{i=N}^{W_0-1} P_e^i \cdot C \tag{14}$$

where $$A_0 = \frac{1-p}{1-p^{R+1}} \cdot \frac{1}{W_0} \cdot \left(\frac{P_s}{P_e}\right)^{Ns} \tag{15}$$

and $$H_0 = \left(\frac{P_c}{P_e}\right)^{N_0} \tag{16}$$

From (14) we get:

$$\sum_{i=N}^{W-1} P_e^i \cdot C = \sum_{i=N}^{W-1} \left(b_0[i] \cdot a_0[i, N_0]\right) = z_0[N_s, N_0] \tag{17}$$

where b_0 is a 1xW_0 array and a_0 is a W_0xW_0 array that hold the following values:

$$b_0[i] = P_e^i \quad \text{for } 0 \le i \le W_0 - 1 \tag{18}$$

$$a_0[i,N_0]=C \quad \text{for } N \le i \le W_0-1 \tag{19}$$
$$a_0[i,N_0]=0 \quad \text{for } i < N \tag{20}$$

and z_0 is a W_0xW_0 array that stores the results of the following array multiplication:

$$z_0 = b_0 \cdot a_0 \tag{21}$$

Using (14) and (17) we compute P_0 for N_s and N_0:

$$P_0(N_s,N_0) = A_0 \cdot H_0 \cdot z_0[N_s,N_0] \tag{22}$$

In the array z_0 the lines (rows) represent the successful transmissions and the columns the collided transmissions. We use (16) and (22) for $0 \le N_0 \le W_0-1$ to compute P_0 only for N_s:

$$P_0(N_s) = A_0 \cdot \sum_{N_0=0}^{W_0-1} z_0[N_s,N_0]\cdot h(N_0,1) \tag{23}$$

and
$$P_0(N_s) = A_0 \cdot x_0[N_s] \tag{24}$$

where
$$x_0 = f_0 \cdot h_0 \tag{25}$$

$f_0 = z_0$ and h_0 is an array W_0x1 that hold the following values

$$h_0(i,1) = \left(\frac{P_c}{P_e}\right)^i \quad \text{for } 0 \le i \le W_0-1 \tag{26}$$

x_0 is an 1xW_0 array that hold the result of the array multiplication of f_0 and h_0 (the result of this multiplication is an array of W_0x1).

The probability that a transmitted packet is successfully transmitted from stage 1 after suffering N_1 transmissions (in stages 0 and 1) is:

$$P_1(N_s,N_1) = A_1 H_1 \sum_{v=0}^{N_s}\sum_{k=0}^{N_1}\left(\sum_{i=v+k}^{W_0-1} P_e^i C_0 \sum_{i=N_s+N_1-v-k}^{W_1-1} P_e^i C_1\right) \tag{27}$$

where
$$A_1 = \frac{(1-p)p}{1-p^{m+1}}\cdot\frac{1}{W}\cdot\frac{1}{2W}\cdot\frac{P_s^{N_s}}{P_e^{N_s}} \tag{28}$$

$$H_1 = \left(\frac{P_c}{P_e}\right)^{N_1} \tag{29}$$

$$C_0 = \frac{i!}{v!k!(i-v-k)!} \tag{30}$$

$$C_1 = \frac{i!}{(N_s-v)!(N_1-k)!(i-(N_s-v)-(N_1-k))!} \tag{31}$$

Following the above equations (18)-(20) we create and fill the b_1, a_1 arrays with the corresponding values for stage 1. From (27) we get:

$$\sum_{i=N_s+N_1-v-k}^{W_1-1} P_e^i C_1 = \sum_{i=N_s+N_1-v-k}^{W_1-1}\left(b_1[i]\cdot a_1[i,N_1-k]\right) = z_1[N_s-v,N_1-k] \tag{32}$$

$$P_1(N_s) = A_1 \sum_{v=0}^{N_s} x_0[v] \sum_{k=0}^{N_1} z_1[N_s-v,N_1-k]\cdot h_1[k,1] \tag{33}$$

$$P_1(N_s) = A_1 \sum_{v=0}^{N_s} x_0[v]\cdot f_1[N_1-v] \tag{34}$$

In (34) there is a sum of two convoluted functions that can be computed using convolution operation. Convolution (denoted with *) is the same operation as multiplying the polynomials whose coefficients are the elements of x_0 and f_1:

$$x_1 = x_0 * f_1 \tag{35}$$

where x_1 is an array of 1x(*length*(x_0)+*length*(f_1)-1) elements

The P_1 now can be easily computed using (34) and (35):

$$P_1(N_s) = A_1 \cdot x_1[N_s] \tag{36}$$

From (24) and (36) we conclude that for a given delay D we need only to compute N_s. From (8) and (10) for j=0 we get:

$$N_s = \text{round\_int}\left(\frac{D-T_s}{T_s+(P_c/P_s)T_c+(P_e/P_s)\sigma}\right) \tag{37}$$

The model gets complex in higher stages but the application of convolution operation keeps the model simple. The corresponding x_j for stage j is:

$$x_j = x_0 * x_1 * x_2 \ldots * x_{j-1} * f_j \tag{38}$$

The most computer time expensive component is the equation (12) which is inside triple nested loops:

$$0 \le N_s \le W_j - 1, \tag{39}$$
$$0 \le N_j \le W_j - 1, \tag{40}$$
$$N_s + N_j \le i \le W_j - 1 \text{ and } 0 \le j \le m \tag{41}$$

4.1 Analytical Model

Let us now give a general form of computing delay distribution P as in the preceding section we provided the background of our model. For a pair of values $k_s = k_1$ and $k_c = k_2$ (where $k_s = N_s$ and $k_c = N_j$) equation (12) gives the same result as for values $k_s = k_2$ and $k_c = k_1$. This property can produce a two dimensional array with values that are symmetrical across primary diagonal. Thus we can reduce the number of loops as the (39) becomes $0 \le k_s \le (W_j-1)/2$ and the (40) becomes $k_s \le k_c \le W_j - 1$, with this way only the half values of the symmetrical array will be computed and other half will be created by "copy" and "paste".

A special case in the computation of (12) is when $k_s = k_c$. Let Y_{k_S} be the result of (12) for a value k_s (initial value k_s =0) then the result of Y_{k_S+1} for k_s+1 is computed as following:

$$Y_{k_S+1} = Y_{k_S} \cdot \frac{(2k_s-1)(2k_s)}{k_s^2} \quad (42)$$

Let $V_{k_C} = Y_{k_S}$ be the result of (42) for $k_c = k_s$ then the result of V_{k_C+1} for k_{c+1} keeping k_s stable is given by:

$$V_{k_C+1} = V_{k_C} \cdot \frac{(k_s+k_c+1)}{k_c+1} \quad (43)$$

Let $C_i = V_{k_C}$ be the result of (43) for $i = k_s + k_c$ then we can compute C_{i+1} for i+1 keeping k_s and k_c stable:

$$C_{i+1} = C_i \cdot \frac{(i+1)}{(i+1-k_s-k_c)!} \quad (44)$$

Let a_{j,k_S} be m two-dimensional arrays (W_j x W_j) to store the following numbers:

$$a_{j,k_S}[i,k_c] = C_i \quad (45)$$

for $0 \le k_s \le (W_j-1)/2$, (46)

$k_s \le k_c \le W_j - 1,$, (47)

$k_s + k_c \le i \le W_j - 1$ and $0 \le j \le m$ (48)

Let y_j be m one-dimensional arrays (1x($W_j/2$)) to store the following numbers (as presented in (42):

$$y_j[i] = 1 \text{ for } i=0 \quad (49)$$

$$y_j[i+1] = y_j[i] \cdot (2k_s-1)(2k_s)/k_s^2$$

$$\text{for } 0 \le k_s \le (W_j-1)/2 \quad (50)$$

Let b_j and h_0 be m arrays that store the following values:

$$b_j[i] = P_e^i \quad \text{for } 0 \le i \le W_j - 1,\ 0 \le j \le m \quad (51)$$

$$h_j[i,1] = (P_c/P_e)^i \quad \text{for } 0 \le i \le W_j - 1,\ 0 \le j \le m \quad (52)$$

Let t_a be a temporary array of 1xW_j elements that store the results of the following array multiplication:

$$t_a = b_j \cdot a_{j,k_S} \quad (53)$$

and z_j be m temporary arrays of W_j x W_j elements that store the following array values:

$$z_j[k_s, k_s:le] = z_j[k_s:le, k_s] = t_a[1, k_s:le]$$

$$\text{for } 0 \le k_s \le W_j - 1 \quad (54)$$

where $k_s:le$ means from line (comumn) k_s to the last line (comumn) le. With (54) we generate the whole symmetrical array.

Let f_j be R+1 arrays of W_j elements that store the values of the following array multiplication:

$$f_j[k_s] = z_j[k_s, 0:le] \cdot h_j$$

$$\text{for } 0 \le k_s \le W_j - 1,\ 0 \le j \le m \quad (55)$$

and $f_{m+u+1} = f_{m+u}$ for $m < m+u+1 \le R$ (56)

where u=0,1,2,… is the remaining number of retransmissions trials in order to reach R.

We extend (15) to cover all stages:

$$A_j = \frac{(1-p)p^j}{1-p^{R+1}} \frac{1}{2^{j(j+1)/2} W^{j+1}} \left(\frac{P_s}{P_e}\right)^{N_S} \quad 0 \le j \le m \quad (57)$$

$$A_{m+u+1} = \frac{(1-p)p^{m+u+1}}{1-p^{R+1}} \frac{1}{2^{m(m+3)/2} W^{m+2}} \left(\frac{P_s}{P_e}\right)^{N_S}$$

$$\text{for } m < m+u+1 \le R \quad (58)$$

The general form to compute P_j is as following:

$$P_j(N_s) = A_j \cdot x_j[N_s] \quad \text{for } 0 \le j \le m \quad (59)$$

$$P_{m+u+1}(N_s) = A_{m+u+1} \cdot x_{m+u+1}[N_s] \quad (60)$$

where: $x_j = f_j$ for $j=0$ (61)

$$x_j = x_{j-1} * f_j \quad \text{for } 1 \le j \le m \quad (62)$$

$$x_{m+u+1} = x_{m+u} * f_m \quad \text{for } m < m+u+1 \le R \quad (63)$$

Now the probability P of (6) can be computed as:

$$P(D) = \sum_{j=0}^{R} P_j(N_s) \quad (64)$$

where $D = N_s\left(T_s + T_c(P_c/P_s) + \sigma(P_e/P_s)\right) + T_s$ (65)

5. Model Validation and Evaluation

In order to validate our model we compare simulative to analytical results. The parameter values used for both simulation and analytical results follow the values specified for the Direct Spread Sequence Spectrum (DSSS) employed in the IEEE 802.11b standard and are shown in Table 1.

Table 1. System parameter values

Channel bit rate	1 Mbit/s
Packet Payload	8184 bits
MAC header	224 bits
PHY header	192 bits
ACK	112 bits + PHY header
RTS	160 bits + PHY header
CTS	112bits + PHY header
Propagation delay, δ	1 µs
Slot time, σ	20 µs
SIFS	10 µs
DIFS	50 µs
Minimum W, W_0	32
Number of W sizes, m	5
Short retry limit, R	6

Fig. 1 plots the packet delay distribution (probability vs. delay) for networks of n=5 and n=50 stations. The model is accurate as the analytical curves match the simulation ones. For n=50 in delay times >1.7secs we observe some differences between the simulation and the analytical values in the order of up to $8 \cdot 10^{-5}$. All simulation results are taken with a 95% confidence interval lower than 0.0001. If we zoom Fig. 1 in x-axis from 0.0-0.5secs we get Fig. 2 (note that y-axis is in linear scale). The figure confirms that the model is accurate as the simulative lines closely follow analytical lines.

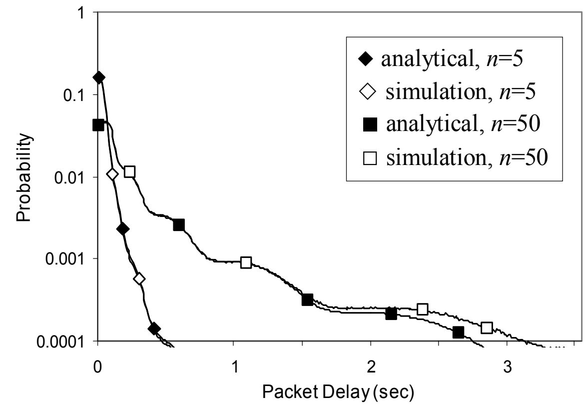

Figure 1. Delay distribution (probability versus delay) for n=5 and n=50.

In order to understand the computational efficiency of our model (equations (42) to (65)), we measured the times required to perform the computation. The most expensive component in computer time is the computation of P_5 (probability of a successfully transmitted packet from stage 5) for W=32 as it requires processing of 1024x1024 element arrays. The computation of P_6 (uses same backoff window size as P_5) is fast as it uses the results of the processed P_5. The P_5 plus P_6 contribute to P from 10^{-6} to $9 \cdot 10^{-5}$ for n=50, from 10^{-6} to $3 \cdot 10^{-5}$ for n=25 and from 0.0 to 10^{-6} for n=5. As this contribution is small, we excluded from the computation of P the computation of P_5 consequently the P_6 in order to reduce the runtimes. We verified that our model computes a single value of the probability P for a given delay D in less than 1.09 secs in a PC Pentium 3GHz using Matlab7 interpreter. The program code for equations (42) to (56) may be executed only once in a case where the parameters W, m, and R stay unchanged (for example, when P values are computed to plot a delay distribution curve), in such a case the model computes a single value of P in less than 0.007 secs. These results confirm the computational efficiency of the model proposed.

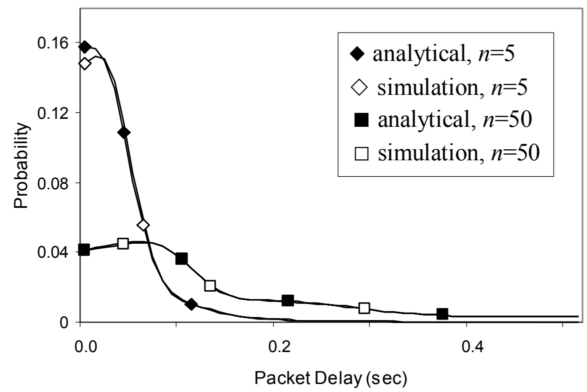

Figure 2. Delay distribution (probability versus delay) for delay 0 to 0.8secs for n=10 and n=50.

Fig. 3 plots the delay distribution at stage 0 for a network of n=25. Each mark in the figure corresponds to one N_s value (the delay for this value is given by equation (65)), so the leftmost mark is for N_s=0, the next for N_s=1 and so on. The figure shows that the simulative results are very close to the analytical ones if not identical validating our model. We used equation (59) for j=0 in order to obtain the analytical curve in the figure.

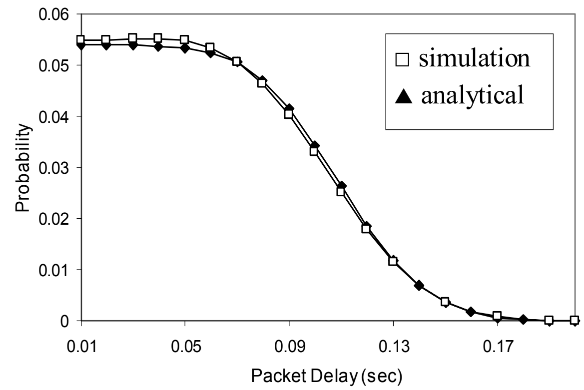

Figure 3. Delay distribution (probability versus delay) at stage 0 for n=25.

Fig. 4 plots the delay distribution (probability versus delay) at stage 0 for n=25. Each spike corresponds to one N_s value and each mark (the simulative and the analytical marks are considered as one) in a spike corresponds to one pair of N_s and N_c values (the delay for this pair of values is given by equation (10) for j=0), so the leftmost spike is for N_s=0, and its leftmost mark is for N_s=0 and N_c=0, the next mark for N_s =0 and Nc=1 and so on. We used equation (22) get the analytical curve. As our model proposes if we add the probabilities in one spike then we will receive the corresponding probability value shown with a mark in figure 3.

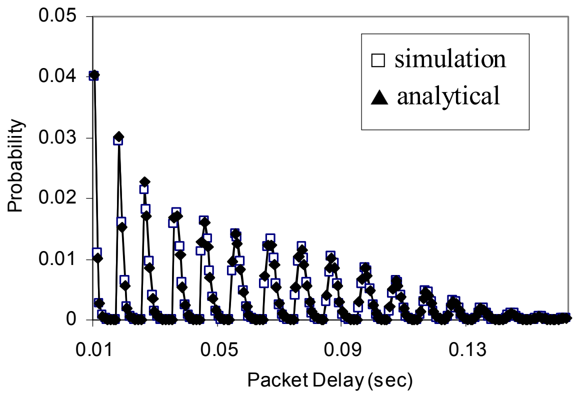

Figure 4. Delay distribution at stage 0 for n=25.

6. Conclusions

In this study we have presented a new analytical model that computes the probability that a single station transmits a packet successfully after suffering a time delay equal to a given value. The key step in our model has been to express a given delay time as a function of the number of transmissions that a packet sees in its backoff procedure until its successful transmission. The model uses this number of transmissions to compute the probability that a packet is successfully transmitted with a delay equal to a given value. The model is simple as uses simple mathematical relations, it is efficient as it requires low computational times, and it is accurate as the analytical results closely match the simulative ones. The model covers only the case of RTS/CTS access scheme.

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